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Determination of a control parameter in a one-dimensional parabolic equation using the method of radial basis functions

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Abstract

In this work, the method of radial basis functions is used for finding the solution of an inverse problem with source control parameter. Because a much wider range of physical phenomena are modelled by nonclassical parabolic initial-boundary value problems, theoretical behavior and numerical approximation of these problems have been active areas of research. The radial basis functions (RBF) method is an efficient mesh free technique for the numerical solution of partial differential equations. The main advantage of numerical methods which use radial basis functions over traditional techniques is the meshless property of these methods. In a meshless method, a set of scattered nodes are used instead of meshing the domain of the problem. The results of numerical experiments are presented and some comparisons are made with several well-known finite difference schemes.

Keywords: Radial basis functions method; Scattered data; Control function; Temperature overspecification; Parabolic equation

1. Introduction

In this paper application of the radial basis functions (denoted RBFs) method to a class of parabolic inverse problems is investigated. Finite difference methods (denoted FDM) are known as the first techniques for solving partial differential equations. Even though these methods are very effective for solving various kinds of partial differential equations, conditional stability of explicit finite difference procedures and the need to use large amounts of CPU time in implicit finite difference schemes limit the applicability of these methods. Furthermore, these methods provide the solution of the problem on mesh points only and the accuracy of the techniques is reduced in non-smooth and non-regular domains. Finite element procedures (denoted FEM) have been used as an alternative method for numerical solution of partial differential equations. This family of numerical schemes is efficient especially for solving problems with arbitrary geometry. But the need to produce a body-fitted mesh in two and three dimensional problems makes this method to be time-consuming and difficult to use. Overall, finite element techniques are highly flexible, but it is hard to obtain results with high order of accuracy. Spectral schemes are accurate but they have less flexibility with the domain of the problem. The use of boundary integral methods avoids body fitted mesh generation for homogeneous

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equations. In fact the boundary element technique (denoted BEM), due to its advantages in dimensional reducibility, requires only a boundary mesh which is considerably more simple to generate than a body-fitted mesh. For an inhomogeneous equation, however, the boundary element method requires a domain node distribution in addition to a boundary mesh [1].

To avoid the mesh generation, in recent years meshless techniques have attracted the attention of researchers. In a meshless (mesh free) method a set of scattered nodes are used instead of meshing the domain of the problem. Some meshless schemes are the element free Galerkin method, the reproducing kernel particle, the local point interpolation, etc. For more descriptions see [2] and references therein.

In the last 20 years, the radial basis functions procedure is known as a powerful tool for the scattered data interpolation problem. The use of radial basis functions as a meshless method for numerical solution of partial differential equations is based on the collocation scheme. Due to the collocation technique, this method does not need to evaluate any integral. The main advantage of numerical procedures which use radial basis functions over traditional techniques is the meshless property of these methods. Since more than 0.7 of the computational effort is used to provide a suitable mesh on the domain of the problem, the meshless property has an auxiliary role.

Radial basis functions are used actively for solving partial differential equations. For example see [3–7]. Also some applications of these functions in solving inverse problems can be found in [8,9].

Consider the inverse problem of finding a source parameter $p(t)$ in the following equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + p(t)u + \varphi(x,t), \quad 0 \le x \le 1, 0 < t \le T, \tag{1.1}$$

with initial condition

$$u(x,0) = f(x), \quad 0 \le x \le 1, \tag{1.2}$$

and boundary conditions

$$u(0,t) = g_0(t), \quad 0 < t \le T, \tag{1.3}$$

$$u(1,t) = g_1(t), \quad 0 < t \le T, \tag{1.4}$$

with the overspecification at a point in the spatial domain

$$u(x_0,t) = E(t), \quad 0 \le t \le T, \tag{1.5}$$

where f, g_0, g_1, φ and E are known functions, and the functions u and p are unknown.

The existence, uniqueness and some applications of this problem are presented in [10–17].

Many methods are developed in the literature for numerical solution of direct parabolic partial differential equations such as finite difference, finite element, spectral, finite volume, boundary element, etc. Although there has been a lot of research into the numerical approximation of direct partial differential equations, there has not been much work for finding the numerical solution of parabolic partial differential equations with overspecified boundary data. The problem of finding the solution of partial differential equations with source control parameter has appeared increasingly in physical phenomena, for example, in the study of heat conduction processes, thermoelasticity, chemical diffusion, and control theory [11–13,15,18–21].

The inverse problem (1.1)–(1.5) can be used to describe a heat transfer process with a source parameter $p(t)$, where (1.5) represents the temperature at a given point x_0 in a spatial domain at time t and u is the temperature distribution. In [22–24] some well-known finite difference techniques are investigated for solving the problem (1.1)–(1.5).

The remainder of the paper is structured as follows.

The radial basis functions are introduced in Section 2. In Section 3, the main problem is reformulated as a direct problem and is solved using the radial basis functions. To present a clear overview of the method, in Section 4 we give an example with analytical solution, and solve it using the radial basis functions. Section 5 ends this paper with a brief conclusion. Also this part gives some directions for future work.

2. Radial basis functions

In this section the RBFs method is defined as a technique for interpolation of the scattered data. Some well-known radial basis functions (RBFs) are listed in Table 1. Let r be the Euclidean distance between a fixed point $x^* \in \mathbb{R}^d$ and

Table 1
Some well-known functions that generate RBFs

Name of function	Definition
Gaussian (GA)	$\phi(r) = \exp(-cr^2)$
Hardy Multiquadric (MQ)	$\phi(r) = \sqrt{r^2 + c^2}$
Inverse Multiquadrics (IMQ)	$\phi(r) = (\sqrt{r^2 + c^2})^{-1}$
Inverse Quadric (IQ)	$\phi(r) = (r^2 + c^2)^{-1}$

any $x \in \mathbb{R}^d$ i.e. $\|x - x^*\|_2$. A radial function $\phi^* = \phi(\|x - x^*\|_2)$ depends only on the distance between $x \in \mathbb{R}^d$ and fixed point $x^* \in \mathbb{R}^d$. This property results that the radial basis function ϕ^* is radially symmetric about x^*. It is clear that the functions in Table 1 are globally supported, infinitely differentiable and depend on a free parameter c.

Let $x_1, x_2, \ldots, x_N$ be a given set of distinct points in $\mathbb{R}^d$. The main idea behind the use of RBFs is interpolation by translation of a single function i.e. the interpolating RBFs approximation is considered as

$$F(x) = \sum_{i=1}^{N} \lambda_i \phi_i(x), \tag{2.1}$$

where $\phi_i(x) = \phi(\|x - x_i\|)$ and λ_i are unknown scalars for $i = 1, \ldots, N$. Assume that we want to interpolate the given values $f_i = f(x_i),\ i = 1, \ldots, N$. The unknown scalars λ_i are chosen so that $F(x_j) = f_j$ for $j = 1, \ldots, N$ which results in the following linear system of equations

$$A\mathbf{z} = \mathbf{f}, \tag{2.2}$$

where $A_{i,j} = \phi_i(x_j)$, $\mathbf{z} = [\lambda_1, \ldots, \lambda_N]$ and $\mathbf{f} = [f_0, \ldots, f_N]$. Since all applicable ϕ have global support, this method produces a dense matrix A. The matrix A can be shown to be positive definite (and therefore nonsingular) for distinct interpolation points for GA, IMQ and IQ by Schoenberg's Theorem [25]. Also using the Micchelli Theorem [26] we can show that A is invertible for distinct sets of the scattered points in the case of MQ.

Although the matrix A is nonsingular in the above cases, usually it is very ill-conditioned i.e. the condition number of A

$$\kappa_s(A) = \|A\|_s \|A^{-1}\|_s, \quad s = 1, 2, \infty, \tag{2.3}$$

is a very large number. Therefore a small perturbation in initial data may produce a large amount of perturbation in the solution. Thus we have to use more precision arithmetic than the standard floating point arithmetic in our computation. For a fixed number of interpolation points the condition number of A depends on the shape parameter c, support of the RBFs and minimum separation distance of interpolation points. Also the condition number grows with N for fixed values of shape parameter c. In practice, the shape parameter c must be adjusted with the number of interpolating points in order to produce an interpolation matrix which is well conditioned enough to be inverted in finite precision arithmetic [5].

Despite research done by many scientists to develop algorithms for selecting the values of c which produce the most accurate interpolation (e.g. see [27,28]), the optimal choice of shape parameter is still an open question.

Generally for a fixed number of collocation points N, smaller values of c produce better approximations, but the matrix A will be more ill-conditioned. Spectral accuracy is obtained in interpolating smooth data using global, infinitely differentiable radial basis functions [29–34].

3. Statement of the problem

In this section the radial basis functions method is implemented for solving the problem (1.1)–(1.5). The first goal is finding the transformations which change Eq. (1.1) to an equation with only one unknown function and then apply the radial basis functions technique on the resulted equation.

Consider the following transformations [19,22]

$$r(t) = \exp\left(-\int_0^t p(s)\mathrm{d}s\right), \tag{3.1}$$

$$w(x,t) = r(t)u(x,t). \tag{3.2}$$

Using the above transformations, Eq. (1.1) is transformed to the equation

$$w_t = w_{xx} + r(t)\varphi(x,t), \quad 0 \le x \le 1, 0 < t \le T, \tag{3.3}$$

with the initial condition

$$w(x,0) = f(x), \tag{3.4}$$

and boundary conditions

$$w(0,t) = r(t)g_0(t), \quad 0 < t \le T, \tag{3.5}$$

$$w(1,t) = r(t)g_1(t), \quad 0 < t \le T. \tag{3.6}$$

It can be seen easily from (3.1) that if $E(t) \neq 0$ then $r(t)$ is obtained in the following form

$$r(t) = \frac{w(x_0,t)}{E(t)}. \tag{3.7}$$

In this transformation the source parameter disappeared and so we can solve the problem (3.3)–(3.7) as a direct problem [35,36].

Now we use the RBFs for discretization of both time and space variables. Let $\Omega = \{(x_i, t_i), 0 \le x_i \le 1, 0 \le t_i \le T, i = 1, \dots, N\}$ be a set of scattered nodes. Then the solution of the problem (3.3)–(3.6) is considered as follows:

$$\tilde{w}(x,t) = \sum_{i=1}^{N} \lambda_i \phi_i(x,t), \tag{3.8}$$

where $\phi_i(x,t) = \phi(\|(x,t) - (x_i,t_i)\|_2)$ for a radial function ϕ and λ_i, $i = 1, \dots, N$ are unknown constants that must be found.

The collocation technique is used for finding unknowns λ_i, $i = 1, \dots, N$. Let

$$\Omega = \Omega_1 \cup \Omega_2 \cup \Omega_3 \cup \Omega_4, \tag{3.9}$$

where

$$\Omega_1 = \{(x_i,t_i), 0 \le x_i \le 1, t_i = 0, i = 1, \dots, N\}, \tag{3.10}$$

$$\Omega_2 = \{(x_i,t_i), x_i = 0, 0 < t_i \le T, i = 1, \dots, N\}, \tag{3.11}$$

$$\Omega_3 = \{(x_i,t_i), x_i = 1, 0 < t_i \le T, i = 1, \dots, N\}, \tag{3.12}$$

$$\Omega_4 = \{(x_i,t_i), 0 < x_i < 1, 0 < t_i \le T, i = 1, \dots, N\}. \tag{3.13}$$

Also we assume $\Omega_i \neq \emptyset$ for $1 \le i \le 4$. Now (3.3)–(3.6) are approximated using (3.8). Thus we have

$$\sum_{i=1}^{N} \phi_i(x_k,t_k)\lambda_i = f(x_k), \quad (x_k,t_k) \in \Omega_1, \tag{3.14}$$

$$\sum_{i=1}^{N} [E(t_k)\phi_i(x_k,t_k) - \phi_i(x_0,t_k)g_0(t_k)]\,\lambda_i = 0, \quad (x_k,t_k) \in \Omega_2, \tag{3.15}$$

$$\sum_{i=1}^{N} [E(t_k)\phi_i(x_k,t_k) - \phi_i(x_0,t_k)g_1(t_k)]\,\lambda_i = 0, \quad (x_k,t_k) \in \Omega_3, \tag{3.16}$$

$$\sum_{i=1}^{N} \left[E(t_k)\frac{\partial}{\partial t}\phi_i(x_k,t_k) - E(t_k)\frac{\partial^2}{\partial x^2}\phi_i(x_k,t_k) - \phi_i(x_0,t_k)\varphi(x_k,t_k)\right]\lambda_i = 0, \quad (x_k,t_k) \in \Omega_4, \tag{3.17}$$

which results in a linear system of equations. Solving this linear system the approximate solution of the transformed problem (3.3)–(3.6) is obtained. The condition number of the resulting linear system depends directly on the shape parameter c. Generally the obtained linear system is ill-conditioned. In [3] a class of numerical methods is proposed for the preconditioning of such matrices.

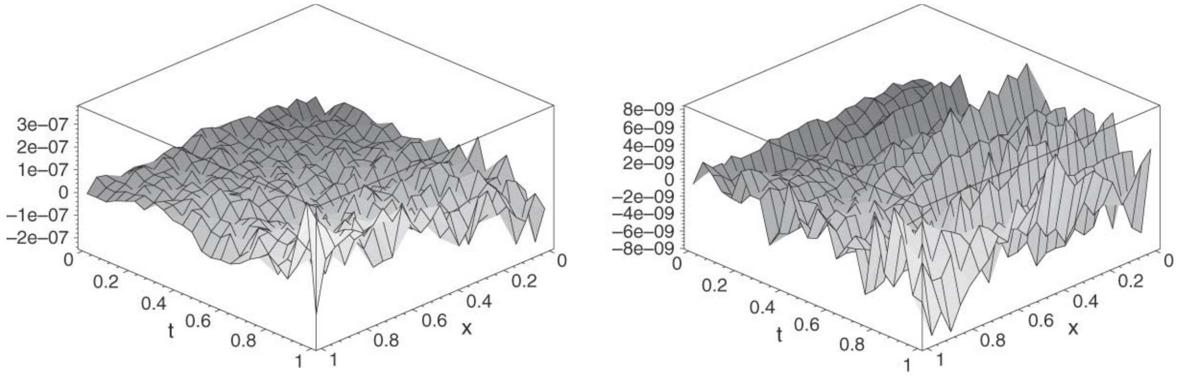

Fig. 1. Plot of error function $u - \tilde{u}$ for a uniform set of collocation points $N = 121$ (left) and $N = 256$ (right) using GA-RBF with $c = 0.1$ and $\delta = 40$.

According to (3.7), an approximation of $r(t)$ is obtained as follows

$$\tilde{r}(t) = \frac{\tilde{w}(x_0, t)}{E(t)}. \tag{3.18}$$

The approximate solution of the main problem (1.1)–(1.5) is

$$\tilde{u}(x, t) = \frac{\tilde{w}(x, t)}{\tilde{r}(t)}, \tag{3.19}$$

and finally the approximate value of $p(t)$ is

$$\tilde{p}(t) = -\frac{\tilde{r}'(t)}{\tilde{r}(t)}. \tag{3.20}$$

4. Test example

To test the efficiency of the method of radial basis functions on the semi-linear inverse parabolic partial differential equation, we present an example. This test is chosen such that its analytical solution is known.

Consider problem (1.1)–(1.5) [22]

$$f(x) = \cos(\pi x) + \sin(\pi x), \tag{4.1}$$

$$g_0(t) = \exp(-t^2), \tag{4.2}$$

$$g_1(t) = -\exp(-t^2), \tag{4.3}$$

$$\varphi(x, t) = (\pi^2 - (t + 1)^2)\exp(-t^2)(\cos(\pi x) + \sin(\pi x)), \tag{4.4}$$

$$E(t) = \sqrt{2}\exp(-t^2), \tag{4.5}$$

and $x_0 = 0.25$, for which the exact solution is

$$u(x, t) = \exp(-t^2)(\cos(\pi x) + \sin(\pi x)), \tag{4.6}$$

and

$$p(t) = 1 + t^2. \tag{4.7}$$

In Fig. 1, the error function $u - \tilde{u}$ is plotted using GA-RBF with $c = 0.1$, $\delta = 40$ (the number of floating point arithmetics) and the set of collocation points $x_i = (i - 1)0.1$ and $t_i = (i - 1)0.1$ for $N = 121$ and $N = 256$. Also the corresponding error function $p - \tilde{p}$ is plotted in Fig. 2.

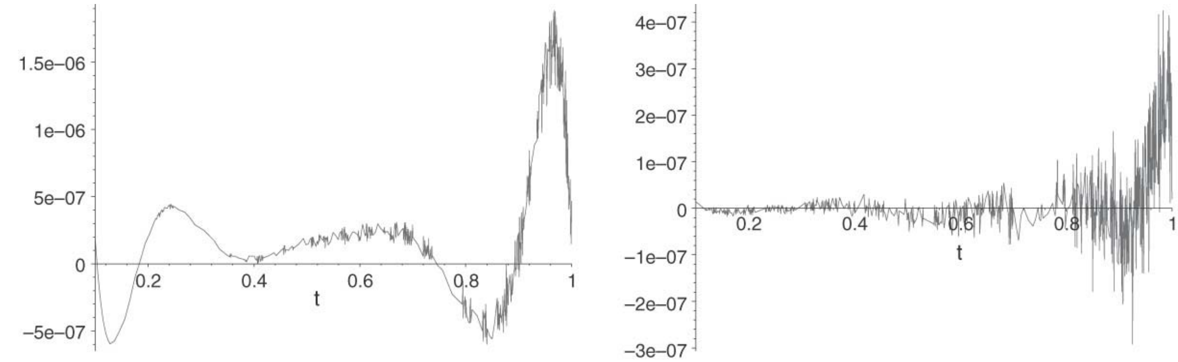

Fig. 2. Plot of error function $p - \tilde{p}$ for a uniform set of collocation points $N = 121$ (left) and $N = 256$ (right) using GA-RBF with $c = 0.1$ and $\delta = 40$.

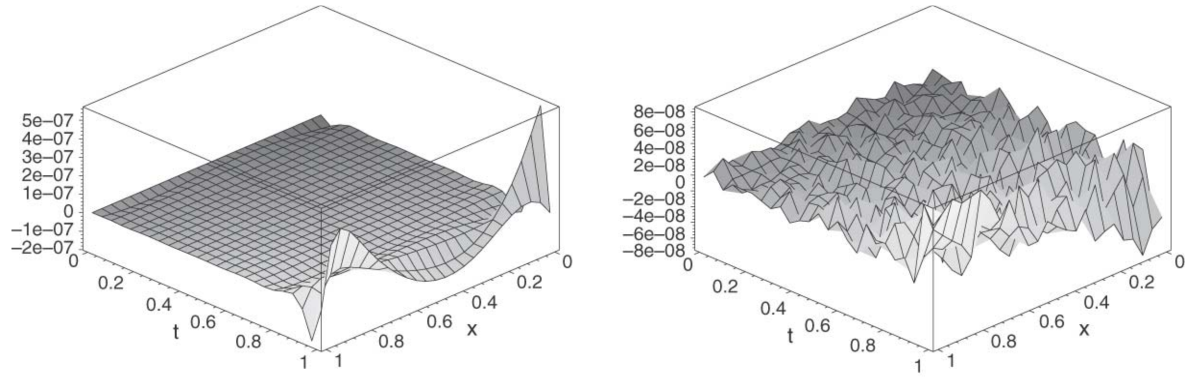

Fig. 3. Plot of error function $u - \tilde{u}$ for a scattered set of collocation points $N = 121$ (left) and $N = 256$ (right) using GA-RBF with $c = 0.1$ and $\delta = 40$.

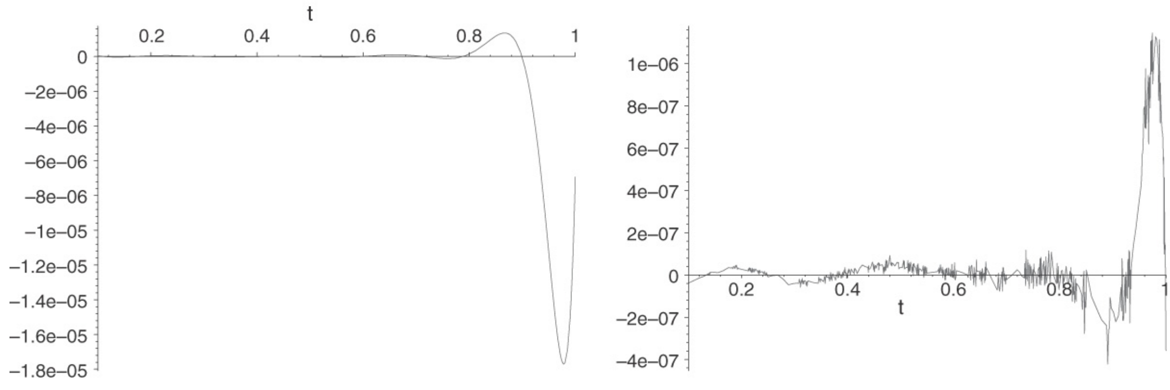

Fig. 4. Plot of error function $p - \tilde{p}$ for a scattered set of collocation points $N = 121$ (left) and $N = 256$ (right) using GA-RBF with $c = 0.1$ and $\delta = 40$.

In Fig. 3, the error function $u - \tilde{u}$ is plotted using GA-RBF with $c = 0.1$, $\delta = 40$ and the set of scattered collocation points for $N = 121$ and $N = 256$. The corresponding error function $p - \tilde{p}$ is plotted in Fig. 4.

In Table 2, some values of the shape parameter c, the corresponding condition number of the matrix which has resulted from discretization of the problem and also the mean square residual error (MSR)

Table 2
Some values of shape parameter c, $\kappa_\infty(A)$, E^2 and E^∞

Shape parameter c	E^2	E^∞	$\kappa_\infty(A)$
0.05	0.2978×10^{-3}	0.4670×10^{-6}	0.3961×10^{44}
0.1	0.1233×10^{-3}	0.2046×10^{-6}	0.2410×10^{44}
1.0	0.2565×10^{-3}	0.1908×10^{-6}	0.1350×10^{33}
10	0.1950×10^{-1}	0.2069×10^{-2}	0.8461×10^{13}
20	0.6256×10^{-1}	0.1878×10^{-1}	0.2307×10^{9}
100	0.1936	0.1466	8738.4021

Table 3
Comparison of the GA-RBFs technique with equidistance collocation points, $N = 256$, $c = 0.05$, $\delta = 40$ and some well-known finite difference methods

x	(1,3) FTCS Error	(1,5) FTCS Error	(3,1) BTCS Error	Crandall Error	GA-RBFs Error
0.05	2.0×10^{-3}	2.9×10^{-6}	4.0×10^{-3}	3.3×10^{-6}	9.7×10^{-8}
0.10	2.0×10^{-3}	2.8×10^{-6}	3.9×10^{-3}	3.3×10^{-6}	2.1×10^{-7}
0.15	2.1×10^{-3}	2.7×10^{-6}	3.8×10^{-3}	3.2×10^{-6}	1.3×10^{-7}
0.20	2.2×10^{-3}	2.7×10^{-6}	3.7×10^{-3}	3.1×10^{-6}	8.6×10^{-8}
0.25	2.3×10^{-3}	2.8×10^{-6}	3.6×10^{-3}	3.1×10^{-6}	1.7×10^{-7}
0.30	2.4×10^{-3}	2.9×10^{-6}	3.6×10^{-3}	3.0×10^{-6}	1.6×10^{-7}
0.35	2.5×10^{-3}	3.0×10^{-6}	3.5×10^{-3}	2.9×10^{-6}	1.0×10^{-7}
0.40	2.5×10^{-3}	3.1×10^{-6}	3.6×10^{-3}	2.9×10^{-6}	8.7×10^{-8}
0.45	2.6×10^{-3}	3.2×10^{-6}	3.7×10^{-3}	2.8×10^{-6}	1.5×10^{-7}
0.50	2.4×10^{-3}	3.3×10^{-6}	3.8×10^{-3}	2.7×10^{-6}	7.8×10^{-8}
0.55	2.3×10^{-3}	3.4×10^{-6}	3.9×10^{-3}	2.8×10^{-6}	5.8×10^{-8}
0.60	2.2×10^{-3}	3.3×10^{-6}	3.9×10^{-3}	2.9×10^{-6}	1.4×10^{-7}
0.65	2.1×10^{-3}	3.1×10^{-6}	4.0×10^{-3}	2.9×10^{-6}	1.4×10^{-7}
0.70	2.0×10^{-3}	3.0×10^{-6}	4.1×10^{-3}	3.1×10^{-6}	1.3×10^{-7}
0.75	2.1×10^{-3}	3.2×10^{-6}	4.1×10^{-3}	3.3×10^{-6}	6.9×10^{-8}
0.80	2.2×10^{-3}	3.3×10^{-6}	4.0×10^{-3}	3.2×10^{-6}	1.3×10^{-7}
0.85	2.3×10^{-3}	3.5×10^{-6}	3.9×10^{-3}	3.4×10^{-6}	1.3×10^{-7}
0.90	2.5×10^{-3}	3.6×10^{-6}	3.8×10^{-3}	3.5×10^{-6}	2.6×10^{-7}
0.95	2.6×10^{-3}	3.7×10^{-6}	4.0×10^{-3}	3.6×10^{-6}	4.8×10^{-7}

$$E^2 = \sqrt{\frac{1}{N}\sum_{i=1}^{N} |u(x_i, t_i) - \tilde{u}(x_i, t_i)|}, \tag{4.8}$$

and the max error

$$E^\infty = \max_{1 \le i \le N} |u(x_i, t_i) - \tilde{u}(x_i, t_i)|, \tag{4.9}$$

are listed for GA-RBF and equidistance collocation points with $N = 121$ and $\delta = 40$.

In Table 3, some approximations of the solution are computed at $t = 1.0$ with $\triangle x = 0.02$, $\triangle t = 10^{-4}$ and $x_0 = 0.25$, using the (1,3) FTCS method, (1,5) FTCS formula, (3,1) BTCS scheme and (3,3) Crandall's technique discussed in [22]. Also the corresponding results obtained using the GA-RBFs method with equidistance collocation points with $N = 256$, $c = 0.05$ and $\delta = 40$ are shown in this table.

In Table 4, the results for $\tilde{p}$ are shown for $\triangle x = 0.02$, $\triangle t = 10^{-4}$ and $x_0 = 0.25$, using the (1,3) FTCS explicit scheme, (1,5) FTCS explicit technique, (3,1) BTCS implicit method and (3,3) Crandall's implicit formula [22]. Also the corresponding results obtained using the GA-RBFs procedure with equidistance collocation points with $N = 256$, $c = 0.05$ and $\delta = 40$ are presented in this table.

Table 4
Comparison of the GA-RBFs method with equidistance collocation points $N = 256$, $c = 0.05$ and $\delta = 40$ and some well-known finite difference schemes

t	(1,3) FTCS Error	(1,5) FTCS Error	(3,1) BTCS Error	Crandall Error	GA-RBFs Error
0.05	4.3×10^{-3}	5.0×10^{-5}	6.1×10^{-3}	5.5×10^{-5}	1.6×10^{-7}
0.10	4.4×10^{-3}	5.0×10^{-5}	6.1×10^{-3}	5.6×10^{-5}	9.3×10^{-8}
0.15	4.3×10^{-3}	5.1×10^{-5}	6.0×10^{-3}	5.5×10^{-5}	1.3×10^{-7}
0.20	4.2×10^{-3}	4.9×10^{-5}	5.9×10^{-3}	5.4×10^{-5}	1.5×10^{-7}
0.25	4.1×10^{-3}	4.8×10^{-5}	5.9×10^{-3}	5.3×10^{-5}	2.2×10^{-7}
0.30	4.1×10^{-3}	4.7×10^{-5}	6.0×10^{-3}	5.4×10^{-5}	8.9×10^{-8}
0.35	3.9×10^{-3}	4.6×10^{-5}	5.8×10^{-3}	5.3×10^{-5}	8.0×10^{-8}
0.40	3.8×10^{-3}	4.5×10^{-5}	5.7×10^{-3}	5.1×10^{-5}	9.1×10^{-8}
0.45	3.9×10^{-3}	4.5×10^{-5}	5.6×10^{-3}	5.0×10^{-5}	4.1×10^{-8}
0.50	3.8×10^{-3}	4.4×10^{-5}	5.5×10^{-3}	4.8×10^{-5}	1.2×10^{-7}

Table 5
Some values of floating point arithmetic δ, E^2, E^∞ using the GA-RBFs method for $c = 0.1$ and equidistance collocation points with $N = 121$

δ	E^2	E^∞
5	1.1908	8.3111
10	2.0087	46.4192
20	0.4666×10^{-1}	0.8803×10^{-2}
30	0.9206×10^{-3}	0.2890×10^{-5}
40	0.1233×10^{-3}	0.2046×10^{-6}
50	0.1123×10^{-3}	0.3930×10^{-7}

To see the dependence of the accuracy of the method and the condition number of the resulting matrix to the number of floating point arithmetic δ, some values of these parameters are shown in Table 5, using the GA-RBFs technique for $c = 0.1$ and the equidistance collocation points with $N = 121$.

5. Conclusion

Radial basis functions were used for solving an inverse semi-linear parabolic equation with a source parameter. The meshless property of the RBFs method is the most important advantage of this scheme over the traditional mesh dependent techniques such as finite difference, finite element and boundary element methods. The mesh free nature of the new technique allows us to solve problems with non-regular geometry. A comparison with some well known finite difference methods for numerical solution of the inverse parabolic problem shows that this technique is accurate. Furthermore, the RBFs method provides a closed form approximation of the solution. Employing a similar procedure for solving the semi-linear parabolic partial differential equation with energy overspecification can be a nice investigation and is the subject of research work proposed by the authors of this paper. In concluding we mention that the RBFs technique can be extended for similar two and three dimensional inverse parabolic problems subject to temperature overspecification.

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